Grades 8–9

Algebra: Equations

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Equation Creation

1

Overview

Mathematical Focus

- Order of operations
- Equivalence
- Arithmetic expressions

Students are introduced to an activity called Equation Creation. In the first several rounds, the student randomly chooses seven numbers from a deck of cards with the numbers 0 through 9 (three of each). They write as many equations as they can with those seven numbers.
For example, if the numbers selected are 0, 2, 3, 5, 5, 8, and 9, some possible equations are:

2 + 3 = 5	85 + 5 = 90	80/2 = 35 + 5
3 + 5 = 8	90 - 5 = 85	

In a second version of Equation Creation the student writes equations that include one variable. Then they must be able to say what value the variable represents.
For example:
80/ x = 35 + 5, x = 2

Preparation and Materials

Before the session, gather the following materials:

- A set of thirty number cards (three each of the whole numbers 0 through 9)
- A set of operation cards (three each of +, -, x, and /; two each of (and); and one = card)
- A watch, clock or other timer

Activity

Part 1: Equation Creation

1. Review the meaning of the term *equation*.

An equation is a number sentence that includes an equal sign. For example:

3 + 4 = 7
3+ 4 = 5 + 2
7 x 3 = 23 - 2
2(10-7)=18/3
4n = 322

These are equations because the expressions on either side of the equal sign are equal.

2. Play a demonstration round of Equation Creation

Have students randomly choose seven of the thirty number cards. The goal for the student is to write as many equations as possible with those seven numbers.

Give the student their operation cards. There should be a total of 17 cards (three each of +, -, x, /; two each of the parentheses, and 1 equal sign). Explain that they may use any of these cards in each of their equations.

Using the 7 number cards and the set of operation cards ask the student to make an equation. Once the student is sure they have an equation, have them record it on a piece of paper.

Using the same 7 number cards, ask the student to make a new equation. Any of the operation cards may be used in each new equation. The student should use the actual cards to make the equation, and then record their equations so they can reuse the cards.

Examples:

Suppose the following number cards are drawn:

0, 0, 1, 2, 4, 5, 8

The following number sentences, as well as others, could be made:

2 x 4 = 8 5 x 4 = 20 8/2 = 4

8/2 = 4 80/40 = 2 2 + 4 = 5 + 1

100/4 = 25

Rules for Equation Creation
-Draw 7 number cards from the deck. (Some numbers may be repeated.)
-Make as many equations as you can using those 7 numbers.
-You can use each of the 7 numbers only once in each equation, but you do not need to use all 7.
-You can use any operations you want.
-You may combine digits to form larger numbers. For example: 5 and 7 to make 75 or 57.

3. Play a new round of Equation Creation.

Return all the number cards to the deck. Choose 7 new cards. Challenge the student to record as many different equations as they can in three minutes, and then check those equations to be sure each one is correct.

Play several more rounds of Equation Creation.

4. Play Equation True or False.

Choose seven number cards. Take turns being the equation creator and the equation checker. The equation creator creates an equation and secretly records whether equation is true or false. The equation checker must then say if the equation is true or false. If it is false, the equation checker changes it to make it true. The equation creator confirms the answer. If there is disagreement, the checker must prove why her answer is correct.

Part 2: Equation Creation with Variables

1. Play Equation Creation with Variables

In this version of Equation Creation have the student write equations that include one variable. Include a variable card (any letter of your choosing) in the deck of cards. Students write as many equations as they can from seven number cards, any operation cards, and one variable card. Have students record each equation on a separate sheet of paper so that they can reuse the cards.

2. **Determine the value of the variable.**

Once students have completed a round of Equation Creation with variables, have them return to the list of equations they recorded and say what the value of each variable in the equations represents.

Play several more rounds of Equation Creation with variables.

Extension

Play Equation Creation with these additional rules:

- players must use at least five of the numbers drawn in every equation; and
- players must use at least one operation on each side of the equation.

From Words to Equations

2

Overview

Mathematical Focus

- Translating words into mathematical symbols
- Identifying the variable in a word problem
- Using variables and equations to represent a situation

Students are presented with simple word problems such as *Natalie is three times as old as her brother Paul*. They begin by describing the situation qualitatively*: who is older—Natalie or Paul?* Students are then asked to answer questions about the problem situation when given a value for one of the variables: *If Paul is 5, how old is Natalie? If Natalie is 27, how old is Paul?* Lastly, students represent the problem situation using variables and an equation such as N=3P or N/3=P. They check their equations by substituting numeric values for the variables.

Preparation and Materials

Before the session, gather the following materials:

- Student Page 1: Situations and Equations

Activity

Part 1: A Process for Moving from Words to Equations

1. **Discuss the statement: "A child's movie ticket costs half the price of an adult ticket."**

   Ask the student: which movie ticket costs more, an adult ticket or a child ticket?
   Once the student can successfully describe the situation (e.g., the adult ticket costs more; the adult ticket costs two times more than the child's ticket), move to the next step.

2. **Answer questions about the situation.**

   If an adult ticket costs $8, how much does a child's ticket cost? [$4]
   If an adult ticket costs $14.50, how much is a child's ticket? [$7.25]
   If a child's ticket costs $5.50, how much is an adult ticket? [$11]
   If a child's ticket costs $4.25, how much is an adult ticket? [$8.50]
   If two child's tickets cost $12, how much is one adult ticket? [$12]
   If two adult tickets cost $18, how much is one child's ticket? [$4.50]

3. **Choose the equation that represents the situation.**

   Explain that you will use the letter C to represent the cost of a child's ticket and the letter A to represent the cost of an adult ticket.

   Present these four equations. Ask the student to choose one equation from the list below that represents the situation.
   $C = 2A$ $C = 1/2A$ $2A = C$ $1/2C = A$

4. **Check the equation by substituting values for the letters.**

   Have the student check the equation they chose by replacing C with $5. What value do you get for A? Does that value make sense? Does the equation make sense?

   One correct equation is $C = 1/2A$. Another correct equation is $2C = A$.

   If your student chooses an equation that does not make sense, help her determine why the equation does not work for the situation. Invite the student to choose a different equation and check it.

Second, check for numerical understanding. For example: if a child's ticket costs $5, how much is an adult ticket?
Third, check for conceptual understanding. For example: write an equation to represent this situation (or choose one from a list of equations).

Part 2: Words to Equations

1. Review the process introduced in Part 1 for moving from words to equations.

Steps for moving from words to equations:

1. Discuss the problem and describe the situation, in other words, tell what the problem is about.

2. Ask and answer questions about the situation using real numbers, for example, if an adult ticket costs $8, how much does a child's ticket cost?

3. Choose or write an equation to represent the situation.

4. Check the equation by substituting values for the letters in the equation.

Use this same process to discuss and evaluate the problem situations on Student Page 1: Situations and Equations. Part 1 of the student page provides additional practice analyzing situations and choosing appropriate equations to represent those situations. In the second part of the student page, students are given an equation and asked to write a corresponding situation. Equations for each of the situations in Part 1 are given below for your reference.

For every human year they are alive, a dog ages 7 years. [dog age = 7* human age; D = 7H, or D/7=H]

An eighth grade student spends 5 hours a week more on homework than a sixth grade student.
[Eighth grade hours on homework = Sixth grade hours + 5; E = S+5, or E-5=S]

In packages of jellybeans there are 3 times as many pink jellybeans as

black jellybeans.
[number of pink = number of black * 3; P = 3B, or P/3 = B]

The perimeter of a square is 4 times the length of one side. [perimeter = 4*length of a side; P = 4S, or P/4 = S]

In football, a team scores 6 points for every touchdown and 3 points for every field goal.
[score = touchdowns * 6 + field goals * 3; S = 6T + 3F]

It costs $3 an hour to rent a row boat plus a $10 non-refundable fee.
[cost to rent boat = 3 * number of hours + 10; C=3H+10]

Extension

Invite students to make up their own situations. For each situation, they should write it in words and then write at least one equation that represents the situation.

Substitution and Equatior 3

Overview

Mathematical Focus

- Using the concept of a balance to represent equivalence
- Using substitution to keep an equation balanced
- Using substitution to rewrite an equation with a simpler, equivalent equation
- Writing equations to represent situations presented pictorially

In the first part of the activity, students are introduced to balance pictures as a representation of equations. Using substitution, students work with problems to make simpler equivalent balance pictures. They record each balance picture as an equation and then solve the equation. In the second part of the activity, tug-of-war provides the context in which students apply substitution to keep the sides of a tug-of-war picture equal. Each time they make a substitution they rewrite the equation to represent the tug-of-war picture and once again, solve the equation. Students are challenge to create and solve their own pan balance and tug-of-war equations.

Preparation and Materials

Before the session, gather the following materials:

- Pan balance scale (optional)
- Coins (optional)
- Student Page 2: Blank Pan Balance Scales,, several copies
- Colored markers (optional)

Notes

If a pan balance scale is available, demonstrate how it is used. Place 10 coins on each pan. The pans should balance. Remove five coins from one pan. The balance should tip toward the heavier pan. Remove five coins from the other pan so the pans balance. Now, add 12 coins to one pan (for a total of 17 in one and 5 in the other). Ask: *How many coins do we need to add to the other side to get the pans to* *balance?* [12] Be sure students understand that for the pans to balance you must always have an equal number of coins on each side.

Activity

Part 1: Equations with a Pan Balance Scale

1. **Explore how pan balance scales work.**

   Introduce the idea of a pan balance scale. (See Notes, above.) Tell the student that rather than use the balance scale for this activity, they will draw balance pictures—that is, pictures that represent a balance scale. Remind students that goal in using the pan balance is to equally weigh each side so that the pan will be balanced. The pan balance is used as a metaphor for thinking about balancing both sides of a mathematical equation.

2. **Write equations to match pictures of bananas, apples, and pineapples on balance scales.**

   On a copy of Student Page 2: Blank Pan Balance Scales, create a balance scale with 10 bananas on one side balanced with 2 pineapples on the other side. Draw another balance scale with 1 pineapple on one side, balanced with 2 bananas and 1 apple on the other side. Tell students to imagine that all the bananas weigh the same amount as each other (and all the pineapples weigh the same, and all the apples weigh the same). Ask students to write an equation that represents each of the pictures.

   [insert graphic of banana/pineapple pan balance]

   10 bananas = 2 pineapples (or 10B = 2P)

   [insert graphic of pineapple/banana/apple pan balance]

   1 pineapple = 2 bananas + 1 apple (or 1P = 2B + 1A)

3. Determine how many bananas are needed to balance 1 apple, based on the balance scale pictures.

Have students to use the balance scale pictures to determine the number of bananas that would balance one apple. Ask: *Can you remove some fruit to make the picture simpler?* Encourage students to use "substitution" to answer this question (i.e., substitute 5 bananas for each pineapple). If possible, have students link the substitution process to the equations they wrote.

3. Work through another pan balance problem.

Draw a balance scale with 14 strawberries on one side; on the other side, 1 banana and 6 grapes. Draw a second balance scale with 1 banana on one side and 20 grapes on the other side. Draw a third balance with 2 apples on one side; on the other side, 14 strawberries and 4 grapes. Have students write equations to match each of the pan balance pictures. Then ask students: *How many grapes does it take to balance one apple?*

Encourage students to develop a plan or strategy for finding the number of grapes it takes to balance one apple. If necessary, give a prompt: *Can you substitute one type of fruit for another to make the picture simpler?*

[insert graphic of strawberry/banana/grape pan balance]

14 strawberries = 1 banana + 6 grapes (14S = 1B + 6G)

[insert graphic of banana/grape pan balance)

1 banana = 20 grapes (1B = 20G)

(insert graphic of apple/strawberry/grape pan balance)

2 apples = 14 strawberries + 4 grapes (2A = 1B + 4G)

4. Have students create and solve their own pan balance problems.

Challenge students to create and solve their own pan balance problems. Students may be able to solve some problems by working with the equations, or they may need to re-write equations to represent the substitutions they made with the pictures.

Part 2: Equation Tug-of-War

1. **Introduce a tug-of-war context for representing and solving equations.**

   Have students write equations that represent the following tug of war situations:

   4 oxen = 5 horses (insert graphics)

   1 elephant = 1 oxen plus 2 horses (insert graphics)

   Ask: *Who will win a Tug-of-War: 1 elephant and 3 horses or 4 oxen?*

   Students may draw pictures and use substitution and/or they may use symbols and use substitution. Encourage them to write equations and relate the equations to the process they use to solve the tug-of-war.

2. **Have students create and solve their own tug-of-war problems.**

   Challenge students to create and solve their own tug-of-war problems. Students may be able to solve some problems by working with the equations, or they may need to re-write equations to represent the substitutions they made with the pictures.

Balancing Equations

4

Overview

Mathematical Focus

- Using the concept of a balance to represent equivalence
- Using substitution to keep an equation balanced
- Using substitution to rewrite an equation with a simpler, equivalent equation
- Writing equations to represent situations presented pictorially

This activity builds on the work that students did in Activity 3: Substitution and Equations. Once again, pan balance pictures are used to represent equations. Students learn to do the same thing to both sides of a picture to make simpler, equivalent equations. Students represent balance pictures as equations and then solve the equations.

Preparation and Materials

Before the session, gather the following materials:

- Pan balance scale (optional)
- Coins (optional)
- Student Page 2, Blank Pan Balance Scales

If a pan balance scale is available, demonstrate how it is used. Place 10 coins on each pan. The pans should balance. Remove five coins from one pan. The balance should tip toward the heavier pan. Remove five coins from the other pan so the pans balance. Now, add 12 coins to one pan (for a total of 17 in one and 5 in the other). Ask: *How many coins do we need to add to the other side to get the pans to balance?* [12] Be sure students understand that for the pans to balance you must always have an equal number of coins on each side.

Activity

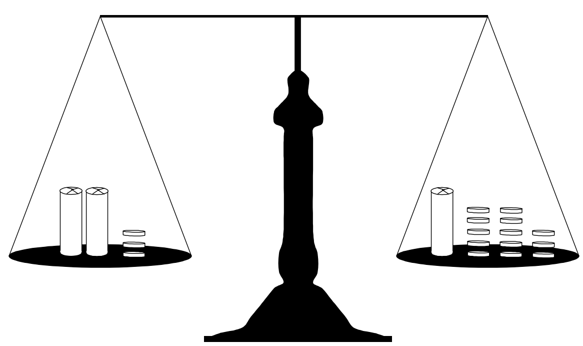
Part 1: The Strange Bank

1. **Introduce the context of a "strange bank" and review the idea of pan balance scales.**

   Tell students that rather than use the balance scale for this activity, they will draw balance pictures—that is, pictures that represent a balance scale.

   Present the following scenario:

   > *There is a very strange bank that does a very strange thing. The people who work there always wrap coins in rolls of weightless paper, but each day they change the number of coins in a roll—so every day the customers have to figure out how many coins are in a roll! But the people who work there always give the customers some clues. On Monday, a customer went into the bank and was given this clue:*

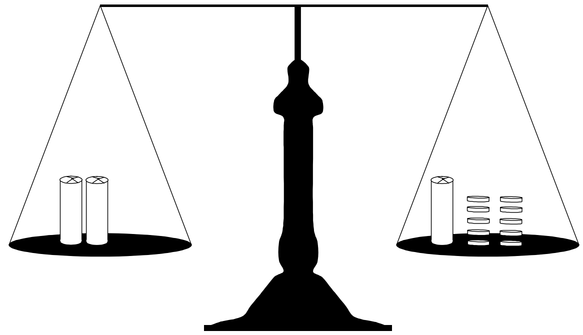


   Explain that this balance picture shows 2 rolls of coins plus 3 loose coins balancing with 1 roll and 13 loose coins.

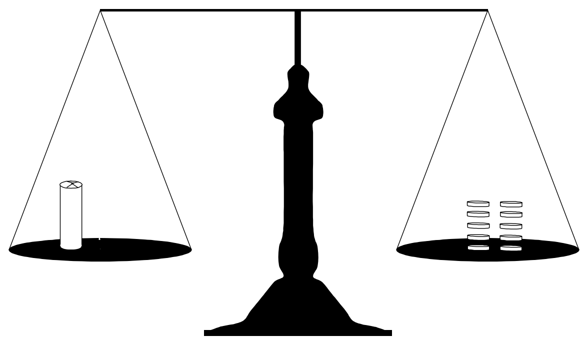
   Ask: *How can we determine the number of coins in a roll?*

   Encourage students to develop a plan or strategy for finding the number of coins in a roll. If necessary, give a prompt: *Let's remove some coins from the pans to make the simplest balance picture we can.* Have students use a copy of Student Page 2: Blank Pan Balance Scales to record their work.

If you remove 3 loose coins from each pan, the balance now looks like this:

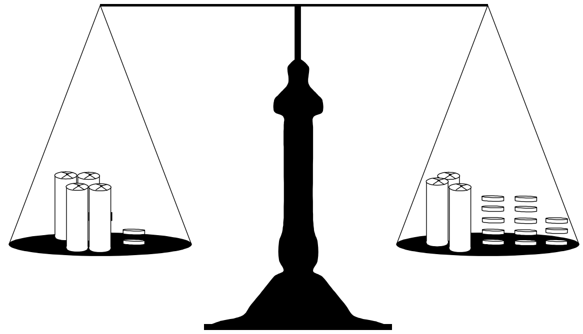


Now, if we remove 1 roll from each pan, the balance looks like this:



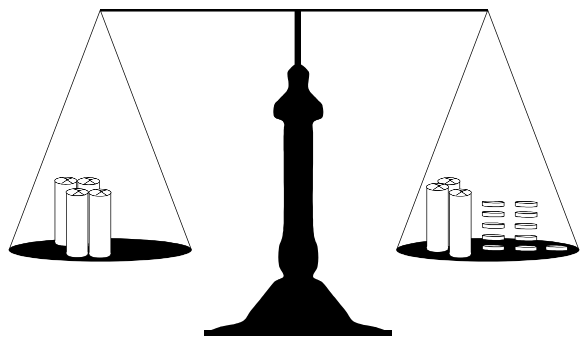
2. **Have students develop a strategy for solving the next problem.**

   Encourage the student to develop a plan or strategy for finding the number of coins in a roll. If necessary, give a prompt: *Let's remove some coins from the pans to make the simplest balance picture we can.*

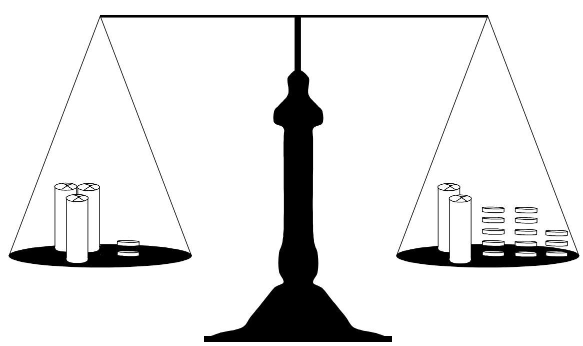


There are many possible answers. Here are a few:

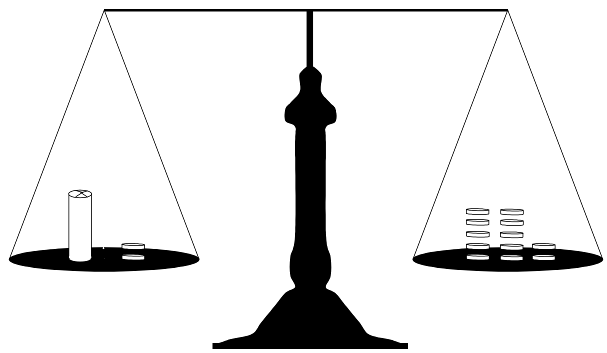
- Remove 2 loose coins from each pan. The new picture would look like this:



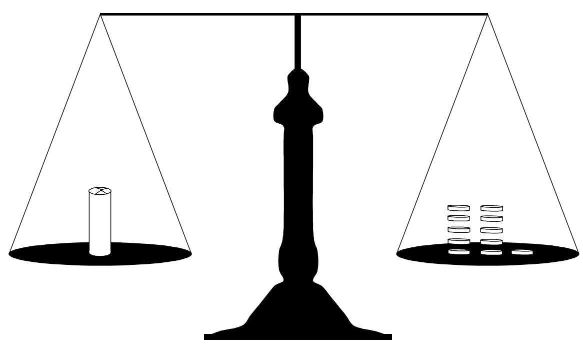
- Remove 1 roll from each pan. The new picture would look like this:



- Remove 3 rolls from each pan. The new picture would look like this:



Encourage students to continue to remove rolls and/or loose coins until they have arrived at the simplest picture, which is:



Part 2: Balance Pictures as Equations

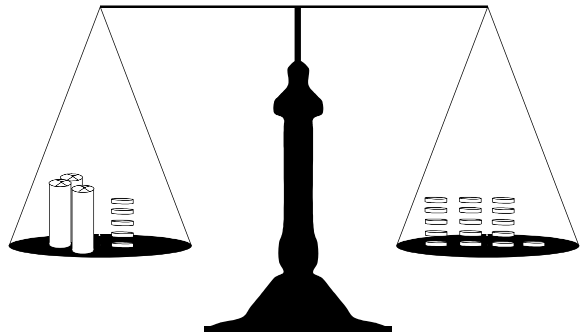
1. Record balance pictures as equations.

Introduce a method for recording balance pictures as equations. For example, the balance picture from step # 5 above can be written as:

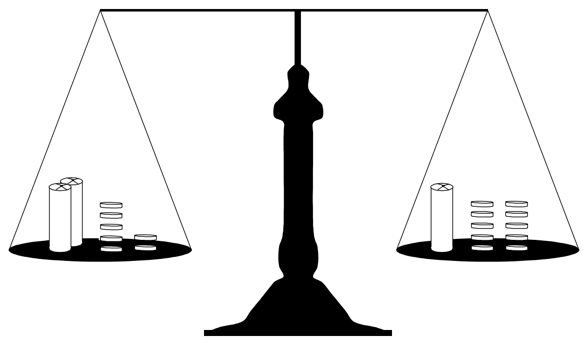
$(4 \times \square) + 2 = (3 \times \square) + 13$

Create a series of balance pictures and have students record the corresponding equation. You may want to use some of the pictures below.

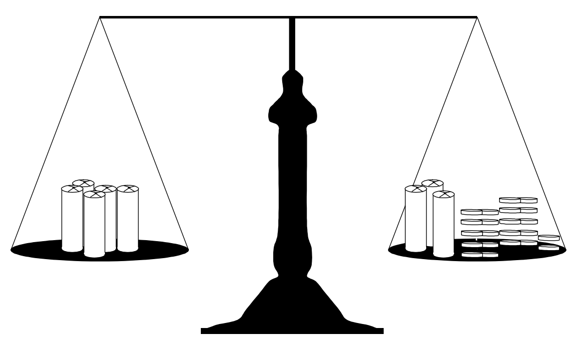
a



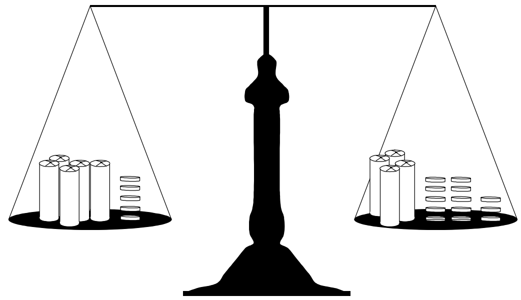
b



c



d



2. Draw balance pictures for equations.

Create some equations and have students draw balance pictures for each equation. Use some of the equations below.

a (6 x ☐) + 4 = 16

b (8 x ☐) + 10 = 50

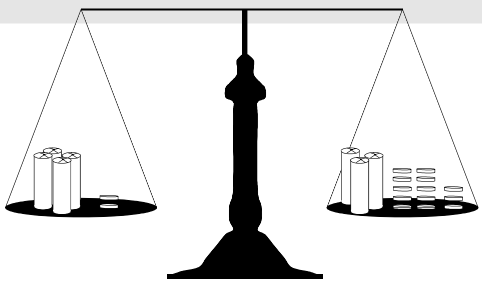
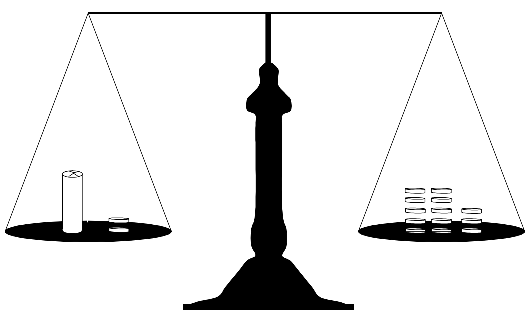
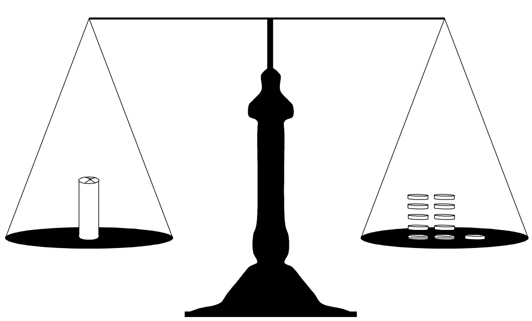
c (10 x ☐) + 7 = (8 x ☐) + 13

d (8 x ☐) + 115 = (5 x ☐) + 127

Part 3: Solve Balance Picture Equations

1. **Solve balance picture equations.**

   Have students solve the balance picture equations they worked with in Part 2. As equations are solved, ask students to record each step in a new corresponding equation. For example:

Balance Picture	Equation
Start with: 	$(4 \times \square) + 2 = (3 \times \square) + 13$
Remove 3 rolls from each pan: 	$(1 \times \square) + 2 = 13$
Remove 2 coins from each pan: 	$1 \times \square = 11$

So, there are 11 coins in a roll.

Check your solution in the original picture and equation.

(4 x 11) + 2 = (3 x 11) + 13

44 + 2 = 33 + 13

46 = 46

Correct!

2. Create and solve additional balance picture equations.

Create more balance pictures for students to solve. For each picture, students should write an equation. Each time they remove coins from a pan, they should rewrite the equation to match the picture. The solution, both picture and equation, should tell how many coins are in a roll.

Extension

Have students create some pan balance problems for you to solve.

Simplifying Sides

5

Overview

Mathematical Focus

- Solving single variable equations
- Applying the concept of balancing the sides of an equation
- Transforming equations

Students are presented with single variable equations such as:
$x + 4 = 5x$ and $5y + 3 = 2y + 15$.
They are asked to recall the balance scale pictures they used in the previous activity. Those pictures and problems involved removing or adding equal amounts to both sides of the scale to a simpler balance picture. In this activity students write down these “moves” using just symbols. Each time they change the equation they do the same thing to the left side and the right side of the equation. They continue to simplify the equation in this way until they are able to arrive at a solution.

Notes

The previous activity, Balancing Equations, should be completed by the student prior to working on this activity.

Activity

1. **Using the image of a pan balance, simplify equations by doing the same operations to both sides.**

   Present two or three equations. For each equation, ask the student to draw or just imagine a pan balance. Each time the student does something to one side of the equations, they should do the same thing to the other side of the equation to keep the scale in balance. At each step, have students rewrite the equation and write a verbal description of how the equation (and balance picture) changed. For example:

Equation	**Description**
Begin with: $4x + 3 = 2x + 11$	
Rewrite as: $2x + 3 = 11$	after subtracting 2x from both sides
Rewrite as: $2x = 8$	after subtracting 3 from both sides
Solution: $x = 4$	after dividing both sides by 2

Equation	**Description**
Begin with: $2n + 9 = 5n + 4$	
Rewrite as: $9 = 3n + 4$	after subtracting 2n from both sides
Rewrite as: $9 = 3n$	after subtracting 4 from both sides
Solution: $n = 3$	after dividing both sides by 3

Equation	**Description**
Begin with: $10p + 4 = 4p + 28$	
Rewrite as: $6p + 4 = 28$	after subtracting 4p from both sides
Rewrite as: $6p + 24$	after subtracting 4 from both sides
Solution: $p = 4$	after dividing both sides by 6

2. **Solve equations by working only with the symbols, performing the same operation on both sides to make the equation simpler.**

   Now present several more equations. This time ask students to solve each equation by working with just the symbols and performing the same operation on both sides, trying to make the equation simpler. Again, ask students to write a verbal description of what they did to the previous equation to get to the simpler equation.

   Sample equations:
   a. $3m + 17 = 4m + 6$
   b. $3r + 4 = r + 14$
   c. $5x = x + 4$
   d. $2t + 20 = 5t + 6$
   e. $b + 12 = 3b + 4$
   f. $5x + 3 = 3x + 15$

3. **Solve equations that can not be represented by pan balance pictures.**

   When the student is comfortable solving equations like the ones above, present a series of equations that are slightly different. Invite the student to solve each equation by doing the same thing to both sides. Some of the equations below include negative numbers which cannot be easily represented using the pan balance image. Although these equations do not lend themselves as well to the pan balance image, the method of solving the equations by doing the same operations to both sides to get a simpler equation is the same.

   Sample equations:
   a. $3x - 2 = x$
   b. $2a - 6 = 0$
   c. $n - 3 = 2n - 6$
   d. $1.5y + 3.5 = 8 + y$
   e. $4/5x = 12$
   f. $5 - 2c = 3c$
   g. $10 + 4x = 2x - 2$
   h. $1/2d = d - 4$
   i. $x/3 = 8 - x$
   j. $2y + 1 = 14 - y$

Extension

Instead of simplifying equations, try *complicating* equations.

Start with any number and write a solution.	x = 10
Multiply both sides by 4.	4x = 40
Add 2x to both sides.	6x = 40 + 2x
Subtract 12 from both sides.	6x – 12 = 28 + 2x
Divide both sides by 3.	2x – 4 = 9 1/3 + 2/3x

Check to make sure 10 is the solution to the final equation.

Now, ask the student to complicate an equation. You should complicate one also. Exchange final equations and solve. Check that the solution you each get is the same as the original number.

Equation Sense

6

Overview

Mathematical Focus

- Developing equation sense
- Solving single variable equations
- Making the best estimate for the value of an unknown

This activity provides students with an opportunity to further develop their *equation sense*, or intuitive notions about solving equations. They begin with an activity in which one number in a single variable equation is covered up. Students are challenged to verbalize questions about the covered number as a way of thinking through what the equation means. For example, in the equation $3 + 17 = 20$, if the 3 is covered up, students ask: *What number plus seventeen equals twenty?* Next students systematically apply a guess and check method to solving equations. They use the data gathered from each guess to inform the next guess until they get to the solution. The process of making a guess and using the information gathered to narrow down a solution develops students' equation sense. In a third activity, students use their sharpened equation sense to solve mystery function machines.

Preparation and Materials

Before the session, gather the following materials:

Strips of blank paper

Coins for covering numbers in an equation

Activity

Part 1: Cover-Up

1. **Ask questions about the numbers covered up in various equations.**

   Write each of the following equations on a slip of paper. Present the equations one at a time with one number in the equation hidden (a coin works well for hiding numbers). Read each equation as a question and ask students to answer the question with a number.

   $3 + 17 = 20$

   Cover the 3 and ask: *What number plus 17 gives 20?*

   $76 – 38 = 38$

   Cover the 76 and ask: *What number minus 38 gives 38?*

   $4 \times 20 = 80$

   Cover the 20 and ask: *Four times what number is 80?*

   $30 \div 15 = 2$

   Cover the 15 and ask: *Thirty divided by what number is 2?*

   Once students are comfortable with equations being read as questions, present a few more equations with one number hidden and this time give *students* an opportunity to read the equation as a question. Ask them to write the question for each equation on a piece of paper and then give the answer.

2. **Determine what number in an equation is covered more than once.**

Write each of the following equations on a slip of paper. Present the equations one at a time with each O covered by a coin. Explain to students that the covered numbers are the same. Read each equation as a question and ask students to answer the question with a number.

O + O = 16

What number plus itself gives 16?

16 = O x O

What number times itself gives 16?

5 + O = 2 + O + 3

Five plus what number equals 2 plus that number plus 3?

[Answer = every number]

O + 3 = O

What number plus 3 equals that same number?

[Answer = no number]

Take turns creating new equations in which the same number is covered more than once. One person creates the equation and covers the numbers. The other person reads the equation as a question and answers the question with a number.

Part 2: Guess, Check, and Improve

Write an equation on a piece of paper for students. You may choose an equation from the list below, or make up your own single-variable equation. The goal for students is to figure out the value of the variable that makes the equation true, without manipulating the equation. Although "Guess, Check, and Improve" is not an efficient or elegant method for solving equations, it is a valuable problem solving exercise for students.

Ask students to begin by guessing a value that might make the equation true. Have them write the guess down, and then substitute that value for the variable in the equation. If the equation is true, ask students how they chose the value they chose. If it is not true, ask students what they think their next guess should be, based on what they have learned from their first guess, and why they think so.

Take turns being the equation guesser. Model your thinking aloud for students when you are guessing the value of the variable. For example:

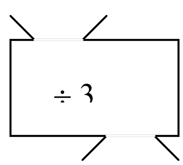
$2x + 3 = 6$

Hmm, I'll guess x = 2 first. If I substitute 2 for x, will the equation be true? 4 + 3 ≠ 6. But I know now that the left side of the equation is a little too big, so I'll substitute 1 for x and see what happens. 2 + 3 ≠ 6. Now the left side of the equation is too small. So, I know that x must be a number between 1 and 2. I'll try substituting 1.5. If I substitute 1.5 for x, I get 3 + 3 which does indeed equal 6.

Part 3: Mystery Function Machines

1. **Ask questions about input and output numbers in a function machine.**

The same process of reading equations as questions can be used with function machines as well. Draw a simple "divide by three" function machine and create an incomplete input/output table such as the one below.



Input	Output
	2
	5
33	

Use the incomplete table to ask equation questions about the missing numbers, for example:

What number divided by three gives two?

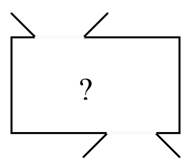
What number divided by three gives five?

Thirty-three divided by three gives what number?

Try a few different function machines and give students a chance to ask (and answer) the equation questions.

2. **Ask equation questions to guess a mystery function machine.**

Think of a mystery function machine but do not tell students what function you are thinking of, for example, you might think of a "times four" machine. Create an incomplete input/output table for your machine such as the one below:



Input	Output
½	2
5	20

Ask equation questions until students guess the mystery function.

What can you do to ½ to get 2?

What can you do to 5 to get 20?

Once students have guessed the mystery function you can still continue to ask equation questions such as the following:

What number times four gives 3?

Four times .25 gives what number?

3. **Create your own function machines.**

Have students create their own function machines and record the questions that are asked and the answers that are given about that machine.

Undoing Equations

7

Overview

Mathematical Focus

- Solving single variable equations
- Using inverse operations
- Using the order of operations when "undoing" equations

This activity provides students with an opportunity to explore equations by working backwards to "undo" the equation. Students start with a given equation and rewrite the equation as a series of operations. They then work through the operations involved in the expression in the reverse order, after which they identify the inverse operation they used for each step. Students take the expression, make it to a function machine, and work backwards and forwards through the function machine to find missing numbers in an input/output table.

Activity

1. **Rewrite equations as series of operations.**

   Start with an equation such as:

   $[7(2x - 3) - 5]/10 = 3$

   Ask students to write down what is operationally happening to the x in this equation. They should write down each operation that they use in a list like the one that follows:

   - Start with X
   - Multiply by 2
   - Subtract 3
   - Multiply by 7
   - Subtract 5
   - Divide by 10
   - The result of these operations on X is 3

2. **Work through the operations involved in the expression in the opposite direction.**

   Work with the same problem but this time, ask students to start with the 3 instead of the X and describe what operations you must do to the 3 to end up with X as the result. Start by writing down the steps listed above in the reverse order. After each step, ask have students ask an equation question that will move them back a step.

   The result of these operations on X is 3

   - Divide by 10

     What do you divide by 10 to get 3? [30]
   - Subtract 5

     From what do you subtract 5 to get 30? [35]
   - Multiply by 7

     What number do you multiply by 7 to get 35? [5]
   - Subtract 3

     From what do you subtract 3 to get 5? [8]

- Multiply by 2

  What do you multiply by 2 to get 8? [4]
- Start with X

  What is X? [4]

3. Write the inverse operations that you used to work backwards.

Have students continue to work with the same equation and list of equation questions. This time, for each equation question, ask students to record the inverse operation they used to work backwards.

The result of these operations on X is 3

- Divide by 10

  What do you divide by 10 to get 3? [30]

  Inverse operation: Multiply 3 X 10

- Subtract 5

  From what do you subtract 5 to get 30? [35]

  Inverse operation: Add 30 + 5

- Multiply by 7

  What number do you multiply by 7 to get 35? [5]

  Inverse operation: Divide 35 ÷ 7

- Subtract 3

  From what do you subtract 3 to get 5? [8]

  Inverse operation: Add 5 + 3

- Multiply by 2

  What do you multiply by 2 to get 8? [4]

  Inverse operation: Divide 8 ÷ 2

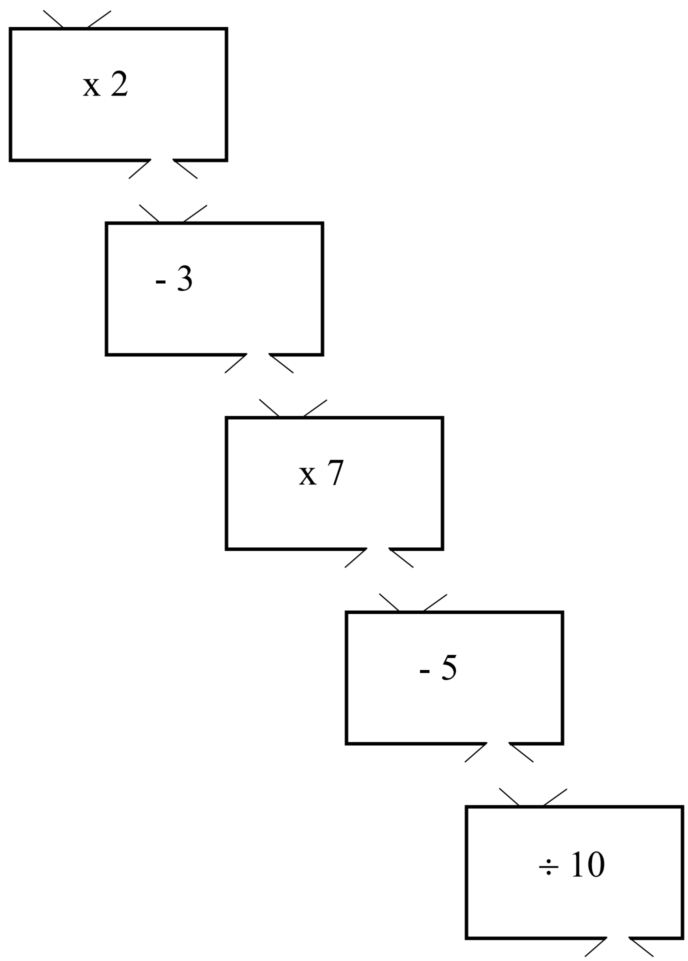
- Start with X

  What is X? [4]

Note: Limitations to this method exist—e.g., 14 – x = 8 (adding back x is not usually used in this method), and 108/x = 9 (multiplying by x is not usually taught), but these equations can be solved by covering-up or guess and check.

4. Turn the equation into a function machine.

Turn the equation into a function machine, leaving off the final result. Find the missing inputs and outputs in a input/output table.



Input	Output
	1/5
4	

	10

5. Work through the four-step process described above to "undo" several more equations.

Have students work through one or more of the problems below using the four-step "undo" process described above. Also invite students to make up their own problems.

$5[(4x + 6)/2] – 1 = 44$

$[(3x/2) – 1]^2 = 25$

$[5x(x^2 + 4)]/2 = 40$

Student Page 1: Situations and Equations

Part 1

For each situation below, write three questions about the situation and answer those questions. Choose an equation that represents the situations. Check the equation by substituting values for the letters.

1. *For every human year they are alive, a dog ages 7 years.*

   Write three questions about the situation and answer them. For example: If a person is 15 years old, how old would they be in dog years? (105 yrs)

   Choose an equation that represents the situation (there may be more than one possible answer):

   D = 7/H   D = 7H   D/7 = H   H = 7D

   Check the equation by substituting values for the letters in the equation.

2. *An eighth grade student spends 5 hours a week more on homework than a sixth grade student.*

   Write three questions about the situation and answer them.

   Choose an equation that represents the situation (there may be more than one possible answer):

   S = E + 5   E = S/5   E = S+5   E-5=S

   Check the equation by substituting values for the letters in the equation.

3. *In packages of jellybeans there are 3 times as many pink jellybeans as black jellybeans.*

   Write three questions about the situation and answer them.

   Choose an equation that represents the situation (there may be more than one possible answer):

   P = 3B   B = 3 * P   B = 3 + P   P/3 = B

   Check the equation by substituting values for the letters in the equation.

4. *The perimeter of a square is 4 times the length of one side.*

Write three questions about the situation and answer them.

Choose an equation that represents the situation (there may be more than one possible answer):

$P * 4 = S$ $S/4 = P$ $P/4 = S$ $P = 4S$

Check the equation by substituting values for the letters in the equation.

5. *In football, a team scores 6 points for every touchdown and 3 points for every field goal.*

Write three questions about the situation and answer them.

Choose an equation that represents the situation (there may be more than one possible answer):

$S = 6T * 3 F$ $S = 6T + 3F$ $6T/3F = S$

Check the equation by substituting values for the letters in the equation.

6. *It costs $3 an hour to rent a row boat plus a $10 non-refundable fee.*

Write three questions about the situation and answer them.

Choose an equation that represents the situation (there may be more than one possible answer):

$C = 3H + 10 H$ $3H – 10 = C$ $C=3H+10$

Check the equation by substituting values for the letters in the equation.

Part 2

Write situations for each of the equations below. Substitute values for the letters in the equation.

$3C + 1 = R$ $D = R * T$ $S/4 = T$

Student Page 2: Blank Pan Balance Scales